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Directional Jones and Stokes vectors

Abstract

Monostatic radar polarimetry requires the simultaneous consideration of electromagnetic waves travelling in opposite directions. The conventional *'Forward Scattering Alignment (FSA)'* convention for setting up a coordinate system for the description of the polarization characteristics of these waves has to be replaced by the *Back Scattering Alignment (BSA)'* convention that makes explicit use of the concept of directional Jones and Stokes vectors based upon the time reversal operation. This concept can be extended also to the bistatic scattering case.

Time reversal and directional Jones/Stokes vectors

Physico-mathematical concepts and techniques which exploit invariants and symmetries for practical applications and basic understanding of physical phenomena are vital for the development of physics and engineering. Besides the fundamental properties of the rotation group (a continuous Lie group) also the discrete groups like inversion of one, two or three space coordinates and time reversal are of great interest and importance. Lateness in recognizing the exact role of symmetries has hindered development and continues to hamstring their applications.

One of the most important discoveries in classical field theory is the Poincaré-invariance of the Maxwell equations. In particular they are invariant with respect to the full symmetry group $O(1,3)$ and not only with respect to the special group $SO(1,3)$. This implies invariance with respect to two Euclidean isometry transformations which preserve the metrics in the separate factors of spacetime:

$$\text{Reflection (parity reversal) } \mathcal{P} : (t, \mathbf{x}) \longrightarrow (t, -\mathbf{x}), \quad \mathbf{x} \in \mathcal{R}^3, \quad t \in \mathcal{R}$$

which changes the orientation of $\mathcal{R}^3$ and

$$\text{Time reversal } \mathcal{T} : (t, \mathbf{x}) \longrightarrow (-t, \mathbf{x}), \quad \mathbf{x} \in \mathcal{R}^3, \quad t \in \mathcal{R}.$$

In this section we are primarily concerned with the time reversal operation and the associated antilinear transformations. Suppose that we have a physical system whose time evolution id described by its trajectory of coordinates and velocities (or momenta) given at each time. Terms such as trajectory, path, worldline and history are used to mean not just the curve or locus traced out in space but the specification also of which locations on the curve are reached at each time.

Time reversal, denoted by $\mathcal{T}$, is described passively by considering a single set of trajectories from the point of view of two different spacetime coordinate systems, related by a reflection of the time coordinate, namely $t \rightarrow \bar{t} = -t$ (and $\mathbf{x} \rightarrow \bar{\mathbf{x}} = \mathbf{x}$). $\mathcal{T}$ is self-inverse $\mathcal{T}^2 = \mathcal{I}$. Given a particle trajectory described by coordinates $\mathbf{x}(t)$ and velocities $\dot{\mathbf{x}}(t)$ which are functions of time t in the first frame, the time reversed trajectory will be obtained by the same coordinate values $\mathcal{T}\mathbf{x}(t) = \mathbf{x}(-t)$ traced in the opposite time direction with reversed velocity values $\mathcal{T}\dot{\mathbf{x}}(t) = -\dot{\mathbf{x}}(-t)$. In this passive description the time-reversed trajectory is traced out forwardly in the new time $\bar{t}$ or towards the 'coordinate past' in the t-system. This latter result is the origin for the term *time reversal.* Since coordinates are just mathematical labels in a passive transformation this does not imply that any physical system evolves towards the past.

If a physical system is time-reversal invariant the new trajectories will be physical trajectories of the original system and active and passive descriptions will be equivalent. The quantities used in the equations describing a time-reversed invariant system must transform under $\mathcal{T}$ in a way that corresponds to the variation in the coordinates. The quantities must in fact *covary* in such a way as to leave the form of the equations unchanged. Such a system is said to have *time-reversal covariance.* Classical electrodynamics are included in this category. Each physical quantity will transform in a specific way under time reversal. The details of each transformation will be determined from the corresponding behaviour of the quantities in terms of which it is defined, or by convention if it is fundamental. The time-reversal operation in quantum systems was first formulated by Wigner [11] in 1932, see also Gottfried [4] and Doughty [3].

The following reasoning (using a linear basis) is convenient. We consider a plane electromagnetic wave propagating in the positive z-direction (indicated by the superscript $^+$)

$$\Re \mathbf{E}^+(t,z) = \Re \begin{bmatrix} E_x^+(t,z) \\ E_y^+(t,z) \end{bmatrix} = \Re \begin{bmatrix} E_x^+ e^{i(\omega t - kz)} \\ E_y^+ e^{i(\omega t - kz)} \end{bmatrix} = \begin{bmatrix} |E_x^+| \cos(\omega t - kz + \phi_x) \\ |E_y^+| \cos(\omega t - kz + \phi_y) \end{bmatrix} \tag{1}$$

Time reversal $\mathcal{T}: t \to -t$ implies

$$\Re \mathbf{E}^+(t,z) \to \Re \mathbf{E}^+(-t,z) =: \Re \mathbf{E}^-(t,z) = \Re \begin{bmatrix} E_x^- e^{i(\omega t + kz)} \\ E_y^- e^{i(\omega t + kz)} \end{bmatrix} = \begin{bmatrix} |E_x^+| \cos(\omega t + kz - \phi_x) \\ |E_y^+| \cos(\omega t + kz - \phi_y) \end{bmatrix} \quad (2)$$

$$= \Re \begin{bmatrix} E_x^{+*} e^{i(\omega t + kz)} \\ E_y^{+*} e^{i(\omega t + kz)} \end{bmatrix} = \Re \begin{bmatrix} E_x^-(t,z) \\ E_y^-(t,z) \end{bmatrix} \quad (3)$$

The Hilbert transform $\mathcal{H}$ of the cosine term (real part) yields the corresponding sine term (imaginary part) which together lead to the exponential or analytic signal representation

$$\cos(x) + i\mathcal{H}(\sin(x)) = e^{ix} \quad (4)$$

A comparison yields the following relation between the complex amplitudes of plane waves propagating in opposite directions which are said to posses the same state of polarization

$$\mathbf{E}^- = \begin{bmatrix} E_x^- \\ E_y^- \end{bmatrix} = \begin{bmatrix} E_x^{+*} \\ E_y^{+*} \end{bmatrix} = \mathbf{E}^{+*}, \quad (5)$$

where a common linear $\{x, y\}$ polarization basis is used.

The Jones vectors $\mathbf{E}^+$ and $\mathbf{E}^-$ are called *directional Jones vectors*, a terminology coined by Graves [5] as early as 1956. Directional Jones vectors that describe the same state of polarization have the same polarization ellipse at a fixed point in space but opposite sense of rotation when viewed from a **common** position. This definition of state of polarization stands in contrast to the IEEE Standard Definitions of Terms for Antennas, 145-1983 [7], in particular to

> **tilt angle (of a polarization ellipse).** When the plane of polarization is viewed from a specified side, the angle measured clockwise from a reference line to the major axis of the ellipse. NOTES: (1) For a plane wave the plane of polarization shall be viewed looking in the direction of propagation. (2) The tilt angle is only defined up to a multiple of π radians and is usually taken in the range $(-\pi/2, +\pi/2)$ or $(0, \pi)$.

For a detailed introduction of directional Jones vectors we refer to Lüneburg [9] and to the forthcoming ONR monograph *Foundations of the Mathematical Theory of Polarimetry.* These vectors are still not universally accepted and are not used for instance in Mott's texbook [10]. A short reference is given, however, in Chapter 5 of the *'Manual of Remote Sensing'* [1], edited by W.-M. Boerner et al. A related concept of Polarization and Phase (PP) vectors has been introduced by Czyz [2].

Historically it is quite interesting how researchers in radar polarimetry have been able to avoid the use of this intriguing concept of time reversal which must be considered as an antilinear operator. Kennaugh [8] considered the dual space of the linear complex 2-dimensional vector space of states of polarizations, i.e. the linear vector space of functionals on that space in terms of the so-called voltage equation. The exclusive utilization of the voltage equation for the calculation of optimal power transfer from an incident wave to a receiving antenna avoids the explicit introduction of the time reversal operation or the use of directional Jones vectors.

Directional Stokes vectors can be introduced along the same lines as directional Jones vectors. The complex conjugation operation $K \equiv^*$ will be replaced by multiplication with a diagonal matrix $K = \text{diag}[1\ 1\ -1]$. Both operations are involutory, $K^2 = I$, i.e., twice the operation leads back to the identity.

Conclusions and suggestions

It is strongly recommended to introduce directional Jones and Stokes vectors following the original suggestion of Graves in 1956 [5]. Graves' original work followed the lines of reasoning initiated by Kennaugh [8] by introducing his pseudo-eigenvalue equations (now called con-similarity equations) derived from the bilinear voltage equation. The physical reason behind all this is the time reversal operation, a nonlinear operator, that has previously been used only in quantum mechanics (Wigner [11]). The fact that it appears also in radar polarimetry is the strong mathematical connection between states of polarization and 2-spin states of quantum mechanics.

The consequences of accepting the nonlinear operation of time reversal in monostatic and bistatic radar polarimetry are not fully understood yet. They may be helpful to solve problems in the retrieval of physical phase differences from scattering matrices and separate them from mathematical artefacts and radar network performance.

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